

1 **Analytic Element Ground Water Modeling as a Research Program (1980-2006)**

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10 research program, history of ground water flow modeling

11 **INTRODUCTION** Every once in a while it is useful to take a step back from the focused  
12 activity of advancing the field of applied geohydrology, and instead try to understand this  
13 activity within its historical and philosophical context. Philosophers of science rarely focus  
14 their attention on the activity of geohydrological modeling, and when they do, they usually look  
15 at the issues of epistemology (or the truth claims) (Oreskes et al. 1994). Meanwhile,  
16 geohydrological researchers and practitioners continue their day-to-day activities, perhaps not  
17 appreciating their role within the larger community.

18 It was the Hungarian philosopher of science Imre Lakatos (1922-1974) who coined the  
19 phrase *research program* as an explanatory concept for understanding progress within science  
20 and engineering communities. A research program is recognized by its hard core (theory),  
21 protective belt (auxiliary assumptions), and heuristic (problem solving machinery) (Lakatos  
22 1970). The hard core of a research program includes the assumptions, methods, and criteria of  
23 acceptance, that form the basis for advancement. For example, the hard core of Newtonian

24 physics is comprised of Newton's three laws of motion plus his law of gravitational attraction.  
25 Additional assumptions that supplement the hard core are referred to as the protective belt. To  
26 illustrate, the hard core of the Copernican Sun centered system was modified to include even  
27 more epicycles than the competing Earth centered Ptolemy system, providing a protective belt  
28 and buying enough time for the Copernican model (almost 100 years) to eventually displace the  
29 incumbent theory. Lakatos devised guidelines for classification of work within a research  
30 program as either a negative heuristic or a positive heuristic. The negative heuristic included  
31 things you should not do, for example, you should not threaten the hard core. The positive  
32 heuristic included things you should do, such as develop the protective belt. An empirically  
33 progressive research program is one that leads to novel predictions or solves new problems, and  
34 the predictions/solutions have been corroborated; otherwise the research program is  
35 degenerating. Lakatos recognized that research programs are dynamic, moving between  
36 progressive and degenerating phases. One should not necessarily abandon a research program  
37 during a degenerating phase.

38 A research program is a different organizing concept than a *research strand* --- the flurry  
39 of activity following the publication of a significant idea or contribution. Schwartz et al. (2005)  
40 discuss the importance of innovative research ideas within the hydrologic sciences by following  
41 the citation trends of influential topical areas. A research strand is usually initiated by a high  
42 impact journal article, goes through a period of high activity, is followed by normal production,  
43 and then eventually fades away. Schwartz et al. (2005) explore evidence of scientific progress  
44 using the idea of *paradigm shift* --- a concept introduced by the American philosopher of science  
45 Thomas Kuhn (1922-1996) --- rather than the Lakatosian *research program*.

46           There is a small but active community of ground water modelers who use an innovative  
 47 solution technique known as the analytic element method (AEM). We will analyze the  
 48 community of AEM researchers using the concepts of Lakatos. To do so, we will describe the  
 49 AEM hard core and protective belt, study its publication output, and assess whether or not the  
 50 AEM research program is in a progressive or degenerative phase.

51 **ANAYLTIC ELEMENT MODELING RESEARCH PROGRAM**       The AEM research  
 52 program includes developers and modelers who use the analytic element method to solve  
 53 regional groundwater flow problems. The AEM is a computational method based on the  
 54 superposition of analytic expressions to represent any three-dimensional or two-dimensional  
 55 vector field. The term analytic is carefully chosen since the flow field is calculated by use of  
 56 analytic functions and is everywhere differentiable, except at some isolated points on (mostly)  
 57 interior boundaries. In this paper the discussion will be restricted to the application of the AEM  
 58 to regional ground water flow modeling. The AEM shares the accuracy of the analytical  
 59 solutions and is intermediate in the complexity of the conceptual model represented (See **Table**  
 60 **1**). See Chapter 8.5 in Fitts (2002) for a lucid comparison of analytic element and the perhaps  
 61 more familiar finite element and finite difference numerical methods.

<b>Table 1. A classification of ground water modeling techniques</b>	<b>Analytical (e.g. Theis solution)</b>	<b>Analytic Element</b>	<b>Numerical (e.g. finite difference, finite element methods)</b>
Solution to the governing differential Equation	Exact	Exact	Approximate
Representation of boundary conditions	Exact	Approximate	Approximate
Suitability for complex hydrogeology	Low	Medium	High

62           The development of the analytic element method came out of the research group at the  
 63 University of Minnesota, Department of Civil Engineering, under the direction of Professor Otto  
 64 D.L. Strack. The research was driven by a practical problem to solve the groundwater/surface

65 water interactions of the 40 mile divide cut canal near the Tennessee-Tombigbee Waterway.  
66 For a historical perspective on the emergence of the AEM theory, the reader is guided to the  
67 autobiographical sketch by Strack (2003a). A very informative historical perspective on AEM  
68 applications is contained in Hunt (2006). It is interesting to visualize the genealogy of the  
69 analytic element research program as a “family tree”, starting with Otto Strack (**Figure 1**).  
70 Leadership at degree granting institutions is highlighted. The branches of the “tree” show the  
71 historical growth of the AEM research program.

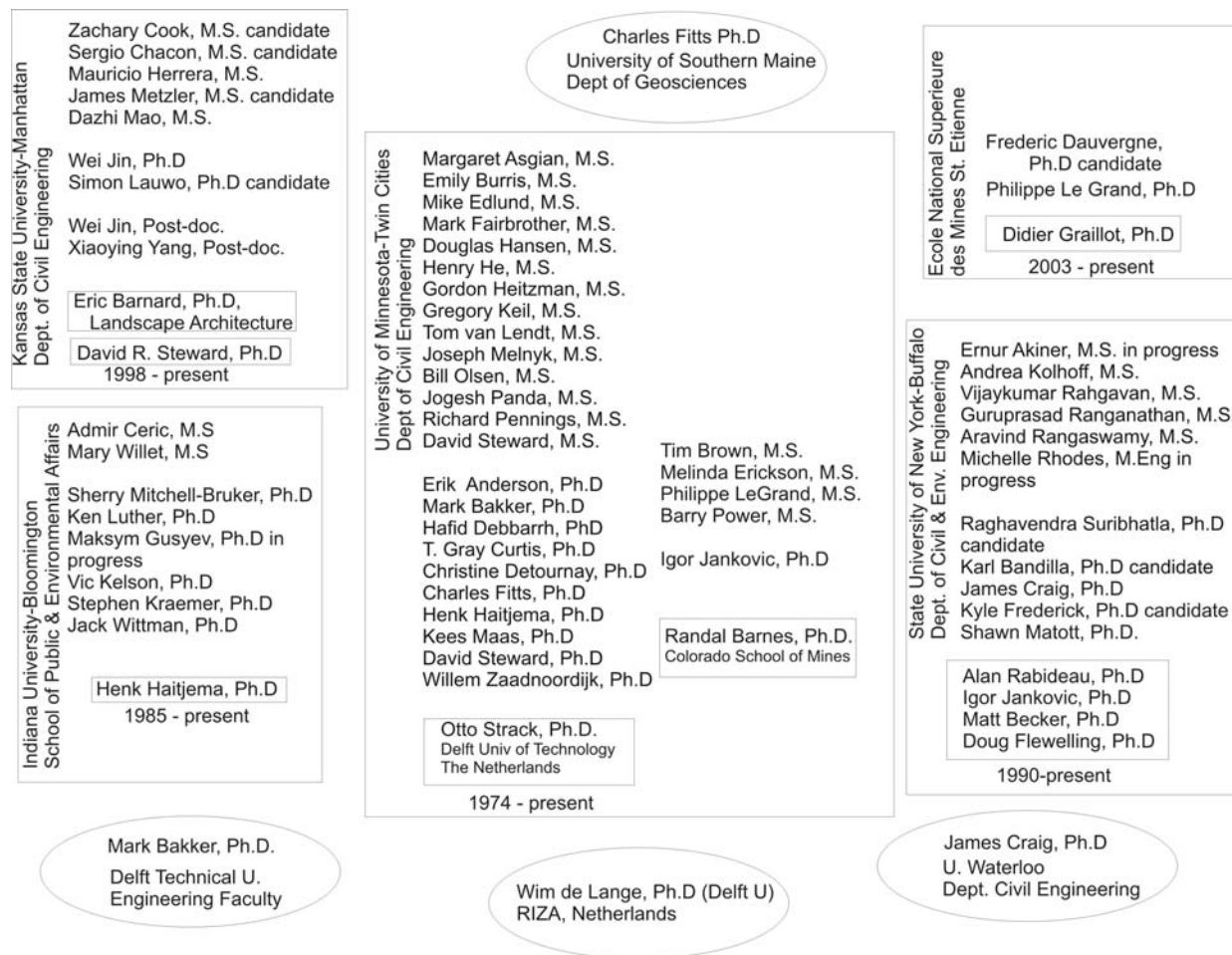
72         The essentials of the theory began to come together in a series of publications in the  
73 1980s: journal articles (Strack and Haitjema 1981a,b), (Strack 1982a), (Strack 1984), (Haitjema  
74 1985), (Strack 1985), (Haitjema 1987), (Haitjema and Kraemer 1988); reports (Strack 1982b),  
75 (Haitjema and Strack 1985); proceedings (Strack et al. 1980); and dissertations (Haitjema 1982),  
76 (Curtis 1983), (Fitts 1985), (Zaadnoordijk 1988). The definitive and comprehensive AEM  
77 publication was the reference book *Groundwater Mechanics* by Strack (1989), while an  
78 application-oriented presentation of the method is given by Haitjema (1995).

79         A formal and mathematical definition of the hard core of the AEM in its current form is  
80 described in Strack (2003b). In broad terms, and in order to be recognized as part of the AEM  
81 research program as presented in this paper, the research should be based on the superposition of  
82 analytic functions in an infinite domain, with these functions representing some expected  
83 hydrologic (ground water) performance, either in two dimensions (Dupuit assumption) or three  
84 dimensions, with internal or external boundary conditions being met exactly or approximately,  
85 satisfying Darcy’s law, maintaining water balance, and showing lineage to the University of  
86 Minnesota group (Strack, Haitjema et al.) either through a direct/indirect education pathway or  
87 through citation/nomenclature. Contributions to the hard core from outside the family tree

88 strengthen the research program. Many in the AEM research community use the Einstein index  
89 convention for documenting analysis (e.g.,  $q_i = -k\partial\phi_i; i = 1,2,3$  ). Those who stick with the hard  
90 core will find their paper or model recognized by their peers within the AEM research program.

91 The protective belt of the AEM research program includes the auxiliary assumptions and  
92 activities that deflect challenges to the hard core. For example, differences in AEM model  
93 predictions and field observations dictate the evolution of the complexity of the conceptual  
94 model represented. Inhomogeneities in aquifer properties, such as hydraulic conductivity, are  
95 represented in a piece-constant manner in current analytic element models using polygon (2D) or  
96 ellipsoidal (3D) elements. The heuristic of adding inhomogeneities to the model to reduce the  
97 error between model and observed hydraulic heads provides an AEM protective belt. A  
98 situation could be imagined where the modeler persists with the continuum representation of the  
99 aquifer, adding more and more polygon inhomogeneities to better calibrate to observations, not  
100 realizing that a discrete flow model might be more appropriate --- such as when investigating a  
101 carbonate rock aquifer.

102



103

104 **Figure 1 The AEM family tree.**

105 It has been intimated at a past AEM international conference that AEM researchers have  
 106 an almost “cult-like” enthusiasm. Is this a characteristic of a Lakatosian research program? Is  
 107 this passion being translated into a progressive (meaning solving new problems) research  
 108 program? Or are the adherents clinging to a program past its peak and beginning to degenerate?

109 **CITATION METHODS**

Citation data and analysis is used in this paper as a measure of the  
 110 activity of the AEM research program, and to suggest its impact on the field of ground water  
 111 flow modeling. Citation analysis is a recent method used to examine hydrological research  
 112 (Schwartz and Ibaraki 2001) (Schwartz et al. 2005), although is not without its critics (Miller and  
 113 Gray 2002). Citation analysis alone is insufficient as a measure of the progressiveness of a

114 scientific research program. However, citation analysis can provide an imperfect measure of  
115 research activity and impact (e.g., research strands).

116 Citation information was obtained from the Web of Science databases maintained by the  
117 Institute for Scientific Information (ISI), the Dialog Corporation SciSearch database, and from  
118 the Proceedings from each of the International Conferences on the Analytic Element Method  
119 (ICAEM): (Haitjema 1994), (de Lange 1997), (Strack 2000), (Graillet 2003), (Steward 2006).  
120 In order to compare the publications relating to ground water flow modeling techniques, the ISI  
121 database was searched with a strict Boolean requirement. For example, the search might look  
122 like “analytic element” AND (groundwater OR “ground water” OR ground-water). The  
123 presence of the word combinations in titles and abstracts constituted a hit. It is recognized that  
124 this search is not comprehensive, but does provide an objective measure of some comparisons  
125 between modeling techniques. Other information was obtained from individual researchers’  
126 curricula vitae and from public web pages. The literature reviews by Craig (2006) and Hunt  
127 (2006) were valuable resources for reports not included in the ISI web of science.

128 **CITATION RESULTS** The emergence of the AEM is recent in relative comparison to  
129 more mature numerical methods used in ground water modeling, such as finite element method  
130 (FEM) and finite difference method (FDM). The earliest FDM journal article applied to ground  
131 water modeling present in the ISI Web of Science database appeared in 1969 [“finite difference”  
132 AND (groundwater OR “ground water” OR ground-water)]. For finite elements the first ISI  
133 journal reference was in 1971. Boundary elements (BEM), another modeling technique, had its  
134 first ISI journal reference in 1981. And the AEM first ISI journal reference occurred in 1992.  
135 These dates are for comparison purposes since clearly these ISI searches did not accurately  
136 reveal the dates of first publication of any of these other mentioned methods. The core of the

137 AEM as a research program had been established well before this date (1992) as discussed  
 138 above. This is likely true for the other numerical methods as well.

139 The Top Ten analytic element publications are based on a ranking of total number Web  
 140 of Science ISI search for journal article citations and Dialog search of ISI for book citations as of  
 141 October 2006 (see **Table 2**). The two most cited works are the books, Strack's (1989)  
 142 Groundwater Mechanics, and Haitjema's (1995) Analytic Element Modeling of Groundwater  
 143 Flow. In order to minimize the advantage of older publications, the citations per year statistic is

**Table 2. Top Ten AEM publications based on total citations.**

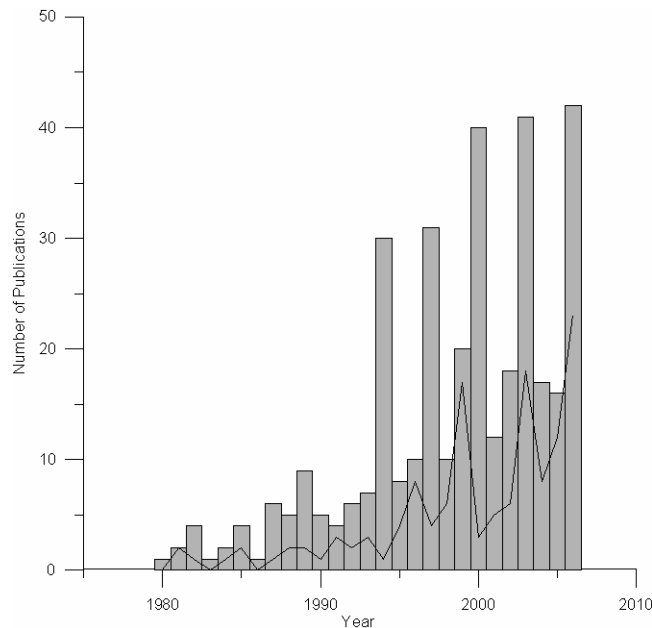
Citation	Times cited	Citations per year	Description
(Strack 1989)	190	11.2	book <u>Groundwater Mechanics</u>
(Haitjema 1995)	61	5.5	book <u>Analytic Element Modeling of Groundwater Flow</u>
(Haitjema 1985)	37	1.8	article <u>Modeling 3D flow in confined aquifers</u>
(Strack 1984)	34	1.5	article <u>3D streamlines in DF model</u>
(Strack and Haitjema 1981b)	32	1.3	article <u>Modeling double aquifer flow using distributed singularities 2.</u>
(Barnes and Jankovic 1999)	29	4.1	article <u>2D flow thru large no. inhomogeneities</u>
(Jankovic and Barnes 1999)	28	4.0	article <u>3D flow thru large no. spherical inhomogeneities</u>
(Haitjema and Kraemer 1988)	22	1.2	article <u>Analytic function for modeling partially penetrating wells</u>
(Bakker and Strack 1996)	20	2.0	article <u>Capture zone delineation in 2D groundwater flow models</u>
(Strack and Haitjema 1981a)	20	0.8	article <u>Modeling double aquifer flow using distributed singularities 1.</u>

144 also shown in **Table 2**. Note that Jankovic and Barnes (1999) and Barnes and Jankovic (1999)  
 145 are poised to move up in ranks in the future based on this impact factor.

146 The number of annual AEM publications is increasing over the period 1980-2006, as  
 147 shown in **Figure 2**. The publications include articles, reports, proceedings, and master theses  
 148 and doctoral dissertations; abstracts and posters are not included. The spikes in the histogram  
 149 correspond to additional publications from the proceedings of the international conferences on  
 150 the AEM (1994, 1997, 2000, 2003, 2006). The spikes in the line graph representing journal  
 151 publications reflect the dates of the AEM special issues (Journal of Hydrology 1999, Ground  
 152 Water 2006). The 2006 year reflects a partial total through October. The AEM wiki wiki

153 bibliography was developed as a compendium of this paper ([www.analyticelements.org/wiki](http://www.analyticelements.org/wiki))  
154 and includes a comprehensive list of AEM publications.

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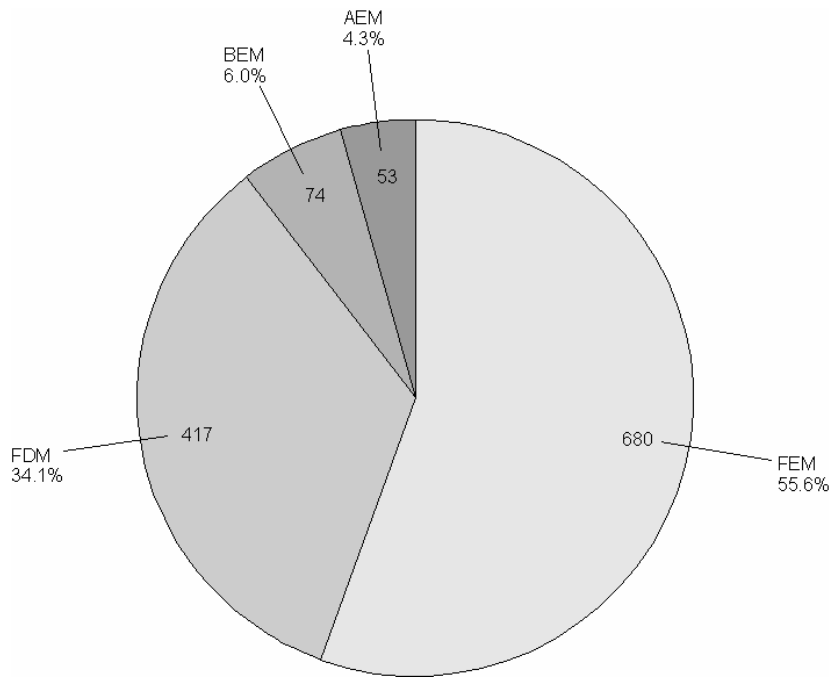
157 **Figure 2 Trend histogram of AEM publications (boxes show total publications; line shows journal articles).**  
158

159 A comparison of total historical ground water modeling publications classified by  
160 numerical modeling technique contained in the IBI database through October 2006 is shown in  
161 **Figure 3.** An example of the Boolean search is [“analytic element” AND (groundwater OR  
162 “ground water” OR “ground-water”)]. The pie chart shows that finite element publications  
163 account for 56% of the total, while finite difference publications account for about 34%,  
164 boundary elements around 6%, and analytic element about 4%.

165 A survey of publicly available analytic element solvers are shown in **Table 3.** Public  
166 domain software gives the model user complete freedom to change, modify, or copy the source  
167 code. Open source software maintains intellectual property and may place some restrictions on  
168 the use and modification of source code. Freeware does not have any cost (royalty free) but the

169 source code is usually not available. A complete list of AEM solvers and modeling systems  
170 (GUIs, input/output support, project organization), including commercial software, is found at  
171 [www.analyticelements.org](http://www.analyticelements.org).

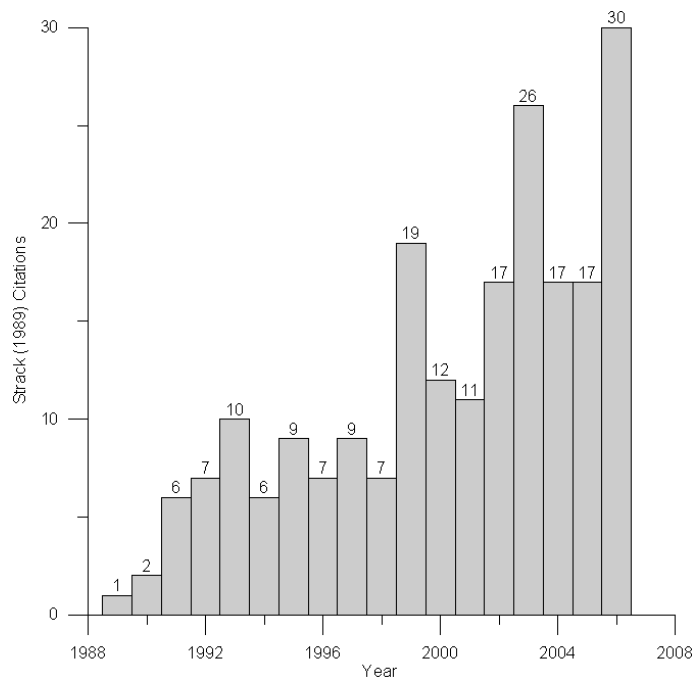
172



173

174 **Figure 3** Ground water modeling publications from IBI database as of October 2006 classified by solution  
175 technique: finite element method (FEM), finite difference method (FDM), boundary element method (BEM),  
176 and analytic element method (AEM).

177 Finally, an analytic element research strand can be demonstrated by following the annual  
178 citations to the Strack (1989) book, as shown in **Figure 4**, based on the ISI Web of Science  
179 database.



180

181 **Figure 4 Analytic element research strand from Strack (1989) book.**

182 **DISCUSSION** The analytic element research program in ground water modeling, while

183 relatively small in comparison to finite element and finite difference research programs, is still in

184 a progressive phase (in a Laktosian sense) after 25 years. Evidence for its progressive state

185 include the expanding publication record, the growing research strand following Strack (1989),

186 the placement of analytic element researchers in academia, and innovative solutions and solvers

187 contributing to the research program.

188 The finite element method is the oldest of the numerical method used in ground water

189 modeling. Given its flexibility in handling complex conceptual models, it dominates the

190 academic community as evidenced in the publication record. The finite difference method, while

191 also quite flexible in representing complex conceptual models, has a strong market share of the

192 publication record.

193            However, it is common understanding that the finite difference method (e.g.,  
 194   MODFLOW) dominates the applied modeling field (Hunt 2006). The popularity of MODFLOW  
 195   is likely due to a number of factors, including its sponsorship by the US Geological Survey and

**Table 3. Publicly available AEM ground water flow solvers.**

<b>Software Name</b>	<b>License</b>
GFLOW	Open source (contact author)
<a href="http://www.haitjema.com">http://www.haitjema.com</a>	
TimSL/TimML	Open source
<a href="http://www.engr.uga.edu/~mbakker/tim.html">http://www.engr.uga.edu/~mbakker/tim.html</a>	
MODAEM	Open source
<a href="http://modaem.sourceforge.net">http://modaem.sourceforge.net</a>	
SPLIT	Open source
<a href="http://www.groundwater.buffalo.edu/software/software.html">http://www.groundwater.buffalo.edu/software/software.html</a>	
PhreFLOW	Open source
<a href="http://www.groundwater.buffalo.edu/software/software.html">http://www.groundwater.buffalo.edu/software/software.html</a>	
3DFlow	Freeware
<a href="http://groundwater.ce.ksu.edu">http://groundwater.ce.ksu.edu</a>	
Bluebird	Open source
<a href="http://www.groundwater.buffalo.edu/software/software.html">http://www.groundwater.buffalo.edu/software/software.html</a>	
SLWL	Freeware (with book Strack 1989)
<a href="http://www.strackconsulting.com">http://www.strackconsulting.com</a>	

196

197 associated quality assurance, the user community comfort with, and understanding of, the finite  
198 difference method, and the rallying of the ground water modeling community behind a single  
199 standard.

200 The emergence of a single community solver in the AEM research community has not  
201 happened (See **Table 3**). The fact that there are a number of solvers is not necessarily a bad  
202 thing. While a single community standard solver might translate into easier user adoption, the  
203 movement toward a single standard might stifle innovation.

204 Arguably, the analytic element models are easier to use since the user (and the GUI  
205 programmer for that matter) does not have to worry about grids or meshes and artificial  
206 boundary conditions. AEM is often used for small projects or as a screening tool for larger  
207 projects (Hunt 2006). The accuracy of analytic element solutions continues to play an important  
208 role in verification and quality assurance of numerical codes.

209 It is probable that the impact of the analytic element method on ground water modeling  
210 will continue to grow. A new generation of researchers are in place as professors in academia.  
211 AEM does outperform numerical methods in some areas, such as including local detail in large  
212 regional models (Simpkins 2006), and as numerical laboratories for study of macro-scale  
213 dispersion (Jankovic et al. 2006).

214 The use of citation records as evidence for the current progressive state of the AEM  
215 research program is open to criticism. For example, expansion of the total publication record  
216 associated with AEM based on citation records might be due to the expansion of the number of  
217 journal outlets of questionable quality. However, this is not the case for the AEM data, since a  
218 vast majority of publications are from four journals of high quality: Water Resources Research,  
219 Ground Water, Journal of Hydrology, and Advances in Water Research.

220           The growth potential of a research program cannot be judged by the publication records  
221 alone, but also by the exciting problems yet to be solved (Miller and Gray 2002). Analytic  
222 element researchers are actively investigating fundamental single phase flow problems, including  
223 transient flow, continuously sloping aquifer properties, and the representation of highly  
224 heterogeneous geology, to name a few.

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226 The citation research relied on the access to the ISI Web of Science through the University of  
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228 Desktop Library. This paper has been reviewed in accordance with the U.S. Environmental  
229 Protection Agency's peer and administrative review policies and approved for publication. The  
230 paper was much improved through reviews by Henk Haitjema, Randy Hunt, and Stella Legarda.  
231 Mention of trade names or commercial products does not constitute endorsement or  
232 recommendation for use

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