Analytic Element Ground Water Modeling as a Research Program (1980-2006)

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INTRODUCTION Every once in a while it is useful to take a step back from the focused activity of advancing the field of applied geohydrology, and instead try to understand this activity within its historical and philosophical context. Philosophers of science rarely focus their attention on the activity of geohydrological modeling, and when they do, they usually look at the issues of epistemology (or the truth claims) (Oreskes et al. 1994). Meanwhile, geohydrological researchers and practitioners continue their day-to-day activities, perhaps not appreciating their role within the larger community.

It was the Hungarian philosopher of science Imre Lakatos (1922-1974) who coined the phrase research program as an explanatory concept for understanding progress within science and engineering communities. A research program is recognized by its hard core (theory), protective belt (auxiliary assumptions), and heuristic (problem solving machinery) (Lakatos 1970). The hard core of a research program includes the assumptions, methods, and criteria of acceptance, that form the basis for advancement. For example, the hard core of Newtonian
physics is comprised of Newton’s three laws of motion plus his law of gravitational attraction. Additional assumptions that supplement the hard core are referred to as the protective belt. To illustrate, the hard core of the Copernican Sun centered system was modified to include even more epicycles than the competing Earth centered Ptolemy system, providing a protective belt and buying enough time for the Copernican model (almost 100 years) to eventually displace the incumbent theory. Lakatos devised guidelines for classification of work within a research program as either a negative heuristic or a positive heuristic. The negative heuristic included things you should not do, for example, you should not threaten the hard core. The positive heuristic included things you should do, such as develop the protective belt. An empirically progressive research program is one that leads to novel predictions or solves new problems, and the predictions/solutions have been corroborated; otherwise the research program is degenerating. Lakatos recognized that research programs are dynamic, moving between progressive and degenerating phases. One should not necessarily abandon a research program during a degenerating phase.

A research program is a different organizing concept than a research strand --- the flurry of activity following the publication of a significant idea or contribution. Schwartz et al. (2005) discuss the importance of innovative research ideas within the hydrologic sciences by following the citation trends of influential topical areas. A research strand is usually initiated by a high impact journal article, goes through a period of high activity, is followed by normal production, and then eventually fades away. Schwartz et al. (2005) explore evidence of scientific progress using the idea of paradigm shift --- a concept introduced by the American philosopher of science Thomas Kuhn (1922-1996) --- rather than the Lakatosian research program.
There is a small but active community of ground water modelers who use an innovative solution technique known as the analytic element method (AEM). We will analyze the community of AEM researchers using the concepts of Lakatos. To do so, we will describe the AEM hard core and protective belt, study its publication output, and assess whether or not the AEM research program is in a progressive or degenerative phase.

**ANAYLTIC ELEMENT MODELING RESEARCH PROGRAM**  
The AEM research program includes developers and modelers who use the analytic element method to solve regional groundwater flow problems. The AEM is a computational method based on the superposition of analytic expressions to represent any three-dimensional or two-dimensional vector field. The term analytic is carefully chosen since the flow field is calculated by use of analytic functions and is everywhere differentiable, except at some isolated points on (mostly) interior boundaries. In this paper the discussion will be restricted to the application of the AEM to regional ground water flow modeling. The AEM shares the accuracy of the analytical solutions and is intermediate in the complexity of the conceptual model represented (See Table 1). See Chapter 8.5 in Fitts (2002) for a lucid comparison of analytic element and the perhaps more familiar finite element and finite difference numerical methods.

<table>
<thead>
<tr>
<th>Table 1. A classification of ground water modeling techniques</th>
<th>Analytical (e.g. Theis solution)</th>
<th>Analytic Element</th>
<th>Numerical (e.g. finite difference, finite element methods)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution to the governing differential Equation</td>
<td>Exact</td>
<td>Exact</td>
<td>Approximate</td>
</tr>
<tr>
<td>Representation of boundary conditions</td>
<td>Exact</td>
<td>Approximate</td>
<td>Approximate</td>
</tr>
<tr>
<td>Suitability for complex hydrogeology</td>
<td>Low</td>
<td>Medium</td>
<td>High</td>
</tr>
</tbody>
</table>

The development of the analytic element method came out of the research group at the University of Minnesota, Department of Civil Engineering, under the direction of Professor Otto D.L. Strack. The research was driven by a practical problem to solve the groundwater/surface
water interactions of the 40 mile divide cut canal near the Tennessee-Tombigbee Waterway. For a historical perspective on the emergence of the AEM theory, the reader is guided to the autobiographical sketch by Strack (2003a). A very informative historical perspective on AEM applications is contained in Hunt (2006). It is interesting to visualize the genealogy of the analytic element research program as a “family tree”, starting with Otto Strack (Figure 1). Leadership at degree granting institutions is highlighted. The branches of the “tree” show the historical growth of the AEM research program.

The essentials of the theory began to come together in a series of publications in the 1980s: journal articles (Strack and Haitjema 1981a,b), (Strack 1982a), (Strack 1984), (Haitjema 1985), (Strack 1985), (Haitjema 1987), (Haitjema and Kraemer 1988); reports (Strack 1982b), (Haitjema and Strack 1985); proceedings (Strack et al. 1980); and dissertations (Haitjema 1982), (Curtis 1983), (Fitts 1985), (Zaadnoordijk 1988). The definitive and comprehensive AEM publication was the reference book Groundwater Mechanics by Strack (1989), while an application-oriented presentation of the method is given by Haitjema (1995).

A formal and mathematical definition of the hard core of the AEM in its current form is described in Strack (2003b). In broad terms, and in order to be recognized as part of the AEM research program as presented in this paper, the research should be based on the superposition of analytic functions in an infinite domain, with these functions representing some expected hydrologic (ground water) performance, either in two dimensions (Dupuit assumption) or three dimensions, with internal or external boundary conditions being met exactly or approximately, satisfying Darcy’s law, maintaining water balance, and showing lineage to the University of Minnesota group (Strack, Haitjema et al.) either through a direct/indirect education pathway or through citation/nomenclature. Contributions to the hard core from outside the family tree
strengthen the research program. Many in the AEM research community use the Einstein index convention for documenting analysis (e.g., \( q_i = -k \partial \phi_i, i = 1,2,3 \)). Those who stick with the hard core will find their paper or model recognized by their peers within the AEM research program.

The protective belt of the AEM research program includes the auxiliary assumptions and activities that deflect challenges to the hard core. For example, differences in AEM model predictions and field observations dictate the evolution of the complexity of the conceptual model represented. Inhomogeneities in aquifer properties, such as hydraulic conductivity, are represented in a piece-constant manner in current analytic element models using polygon (2D) or ellipsoidal (3D) elements. The heuristic of adding inhomogeneities to the model to reduce the error between model and observed hydraulic heads provides an AEM protective belt. A situation could be imagined where the modeler persists with the continuum representation of the aquifer, adding more and more polygon inhomogeneities to better calibrate to observations, not realizing that a discrete flow model might be more appropriate --- such as when investigating a carbonate rock aquifer.
It has been intimated at a past AEM international conference that AEM researchers have an almost “cult-like” enthusiasm. Is this a characteristic of a Lakatosian research program? Is this passion being translated into a progressive (meaning solving new problems) research program? Or are the adherents clinging to a program past its peak and beginning to degenerate?

**CITATION METHODS** Citation data and analysis is used in this paper as a measure of the activity of the AEM research program, and to suggest its impact on the field of ground water flow modeling. Citation analysis is a recent method used to examine hydrological research (Schwartz and Ibaraki 2001) (Schwartz et al. 2005), although is not without its critics (Miller and Gray 2002). Citation analysis alone is insufficient as a measure of the progressiveness of a
scientific research program. However, citation analysis can provide an imperfect measure of research activity and impact (e.g., research strands).

Citation information was obtained from the Web of Science databases maintained by the Institute for Scientific Information (ISI), the Dialog Corporation SciSearch database, and from the Proceedings from each of the International Conferences on the Analytic Element Method (ICAEM): (Haitjema 1994), (de Lange 1997), (Strack 2000), (Graillot 2003), (Steward 2006).

In order to compare the publications relating to ground water flow modeling techniques, the ISI database was searched with a strict Boolean requirement. For example, the search might look like “analytic element” AND (groundwater OR “ground water” OR ground-water). The presence of the word combinations in titles and abstracts constituted a hit. It is recognized that this search is not comprehensive, but does provide an objective measure of some comparisons between modeling techniques. Other information was obtained from individual researchers’ curricula vitae and from public web pages. The literature reviews by Craig (2006) and Hunt (2006) were valuable resources for reports not included in the ISI web of science.

CITATION RESULTS The emergence of the AEM is recent in relative comparison to more mature numerical methods used in ground water modeling, such as finite element method (FEM) and finite difference method (FDM). The earliest FDM journal article applied to ground water modeling present in the ISI Web of Science database appeared in 1969 [“finite difference” AND (groundwater OR “ground water” OR ground-water)]. For finite elements the first ISI journal reference was in 1971. Boundary elements (BEM), another modeling technique, had its first ISI journal reference in 1981. And the AEM first ISI journal reference occurred in 1992. These dates are for comparison purposes since clearly these ISI searches did not accurately reveal the dates of first publication of any of these other mentioned methods. The core of the
AEM as a research program had been established well before this date (1992) as discussed above. This is likely true for the other numerical methods as well.

The Top Ten analytic element publications are based on a ranking of total number Web of Science ISI search for journal article citations and Dialog search of ISI for book citations as of October 2006 (see Table 2). The two most cited works are the books, Strack's (1989) Groundwater Mechanics, and Haitjema's (1995) Analytic Element Modeling of Groundwater Flow. In order to minimize the advantage of older publications, the citations per year statistic is also shown in Table 2. Note that Jankovic and Barnes (1999) and Barnes and Jankovic (1999) are poised to move up in ranks in the future based on this impact factor.

The number of annual AEM publications is increasing over the period 1980-2006, as shown in Figure 2. The publications include articles, reports, proceedings, and master theses and doctoral dissertations; abstracts and posters are not included. The spikes in the histogram correspond to additional publications from the proceedings of the international conferences on the AEM (1994, 1997, 2000, 2003, 2006). The spikes in the line graph representing journal publications reflect the dates of the AEM special issues (Journal of Hydrology 1999, Ground Water 2006). The 2006 year reflects a partial total through October. The AEM wiki wiki
bibliography was developed as a compendium of this paper (www.analyticelements.org/wiki) and includes a comprehensive list of AEM publications.

A comparison of total historical ground water modeling publications classified by numerical modeling technique contained in the IBI database through October 2006 is shown in Figure 3. An example of the Boolean search is [“analytic element” AND (groundwater OR “ground water” OR “ground-water”)]. The pie chart shows that finite element publications account for 56% of the total, while finite difference publications account for about 34%, boundary elements around 6%, and analytic element about 4%.

A survey of publicly available analytic element solvers are shown in Table 3. Public domain software gives the model user complete freedom to change, modify, or copy the source code. Open source software maintains intellectual property and may place some restrictions on the use and modification of source code. Freeware does not have any cost (royalty free) but the
source code is usually not available. A complete list of AEM solvers and modeling systems (GUIs, input/output support, project organization), including commercial software, is found at www.analyticelements.org.

Figure 3  Ground water modeling publications from IBI database as of October 2006 classified by solution technique: finite element method (FEM), finite difference method (FDM), boundary element method (BEM), and analytic element method (AEM).

Finally, an analytic element research strand can be demonstrated by following the annual citations to the Strack (1989) book, as shown in Figure 4, based on the ISI Web of Science database.
DISCUSSION

The analytic element research program in ground water modeling, while relatively small in comparison to finite element and finite difference research programs, is still in a progressive phase (in a Laktosian sense) after 25 years. Evidence for its progressive state include the expanding publication record, the growing research strand following Strack (1989), the placement of analytic element researchers in academia, and innovative solutions and solvers contributing to the research program.

The finite element method is the oldest of the numerical method used in ground water modeling. Given its flexibility in handling complex conceptual models, it dominates the academic community as evidenced in the publication record. The finite difference method, while also quite flexible in representing complex conceptual models, has a strong market share of the publication record.
However, it is common understanding that the finite difference method (e.g., MODFLOW) dominates the applied modeling field (Hunt 2006). The popularity of MODFLOW is likely due to a number of factors, including its sponsorship by the US Geological Survey and

Table 3. Publicly available AEM groundwater flow solvers.

<table>
<thead>
<tr>
<th>Software Name</th>
<th>License</th>
</tr>
</thead>
<tbody>
<tr>
<td>GFLOW</td>
<td>Open source (contact author)</td>
</tr>
<tr>
<td><a href="http://www.haitjema.com">http://www.haitjema.com</a></td>
<td></td>
</tr>
<tr>
<td>TimSL/TimML</td>
<td>Open source</td>
</tr>
<tr>
<td><a href="http://www.engr.uga.edu/~mbakker/tim.html">http://www.engr.uga.edu/~mbakker/tim.html</a></td>
<td></td>
</tr>
<tr>
<td>MODAEM</td>
<td>Open source</td>
</tr>
<tr>
<td><a href="http://modaem.sourceforge.net">http://modaem.sourceforge.net</a></td>
<td></td>
</tr>
<tr>
<td>SPLIT</td>
<td>Open source</td>
</tr>
<tr>
<td><a href="http://www.groundwater.buffalo.edu/software/software.html">http://www.groundwater.buffalo.edu/software/software.html</a></td>
<td></td>
</tr>
<tr>
<td>PhreFLOW</td>
<td>Open source</td>
</tr>
<tr>
<td><a href="http://www.groundwater.buffalo.edu/software/software.html">http://www.groundwater.buffalo.edu/software/software.html</a></td>
<td></td>
</tr>
<tr>
<td>3DFlow</td>
<td>Freeware</td>
</tr>
<tr>
<td><a href="http://groundwater.ce.ksu.edu">http://groundwater.ce.ksu.edu</a></td>
<td></td>
</tr>
<tr>
<td>Bluebird</td>
<td>Open source</td>
</tr>
<tr>
<td><a href="http://www.groundwater.buffalo.edu/software/software.html">http://www.groundwater.buffalo.edu/software/software.html</a></td>
<td></td>
</tr>
<tr>
<td>SLWL</td>
<td>Freeware (with book Strack 1989)</td>
</tr>
<tr>
<td><a href="http://www.strackconsulting.com">http://www.strackconsulting.com</a></td>
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</tr>
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</table>
associated quality assurance, the user community comfort with, and understanding of, the finite
difference method, and the rallying of the ground water modeling community behind a single
standard.

The emergence of a single community solver in the AEM research community has not
happened (See Table 3). The fact that there are a number of solvers is not necessarily a bad
thing. While a single community standard solver might translate into easier user adoption, the
movement toward a single standard might stifle innovation.

Arguably, the analytic element models are easier to use since the user (and the GUI
programmer for that matter) does not have to worry about grids or meshes and artificial
boundary conditions. AEM is often used for small projects or as a screening tool for larger
projects (Hunt 2006). The accuracy of analytic element solutions continues to play an important
role in verification and quality assurance of numerical codes.

It is probable that the impact of the analytic element method on ground water modeling
will continue to grow. A new generation of researchers are in place as professors in academia.
AEM does outperform numerical methods in some areas, such as including local detail in large
regional models (Simpkins 2006), and as numerical laboratories for study of macro-scale
dispersion (Jankovic et al. 2006).

The use of citation records as evidence for the current progressive state of the AEM
research program is open to criticism. For example, expansion of the total publication record
associated with AEM based on citation records might be due to the expansion of the number of
journal outlets of questionable quality. However, this is not the case for the AEM data, since a
vast majority of publications are from four journals of high quality: Water Resources Research,
The growth potential of a research program cannot be judged by the publication records alone, but also by the exciting problems yet to be solved (Miller and Gray 2002). Analytic element researchers are actively investigating fundamental single phase flow problems, including transient flow, continuously sloping aquifer properties, and the representation of highly heterogeneous geology, to name a few.

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Mention of trade names or commercial products does not constitute endorsement or recommendation for use.

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